Broadcast with Hitch-hiking in Wireless Ad-Hoc Networks

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Most results appeared in MASS 2005.

Broadcast in Wireless Networks

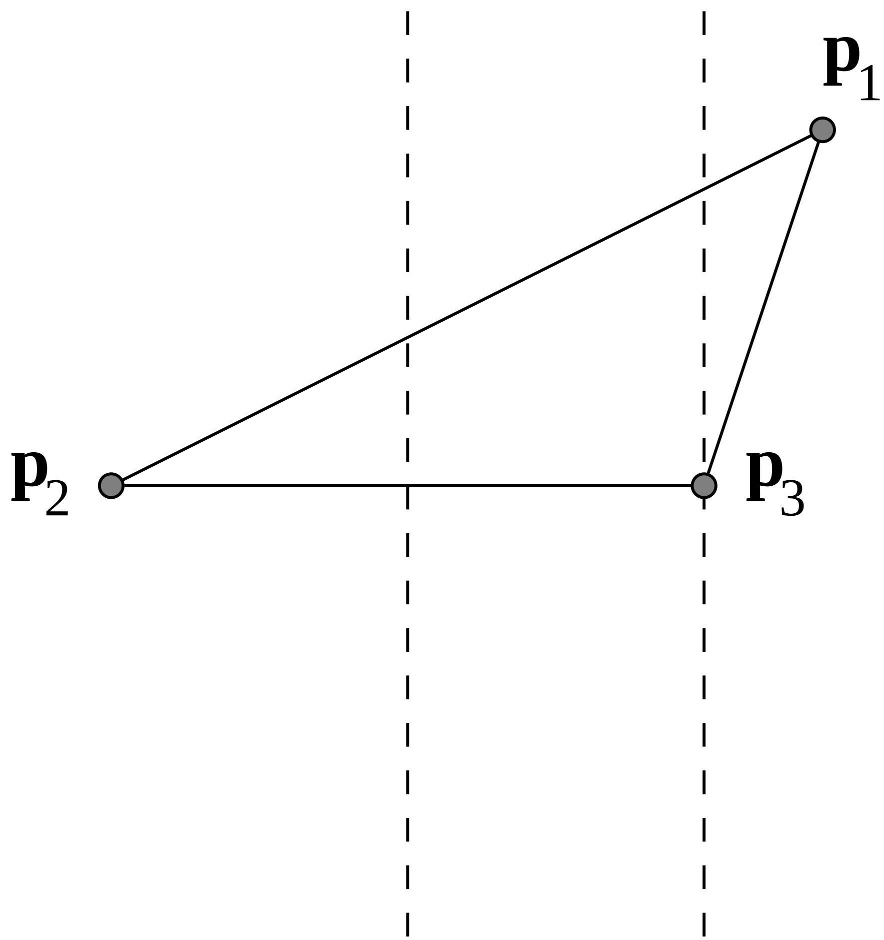

Figure 1: Reducing energy consumption through relaying.

Broadcast in Wireless Networks

Input: $G = (V, E, c)$ weighted directed graph on network nodes with a *power requirement* function $c : E \to R^+$ defined on the arcs, and a root $r \in V$

Output: A *power assignment* function $p : V \to R^+$. A directed arc (u, v) is *supported* by p if $p(u) \geq c(u, v)$. The supported subgraph must contain a path from r to every other node.

Objective: Minimize $\sum_{v \in V} p(v)$

Broadcast in Wireless Networks

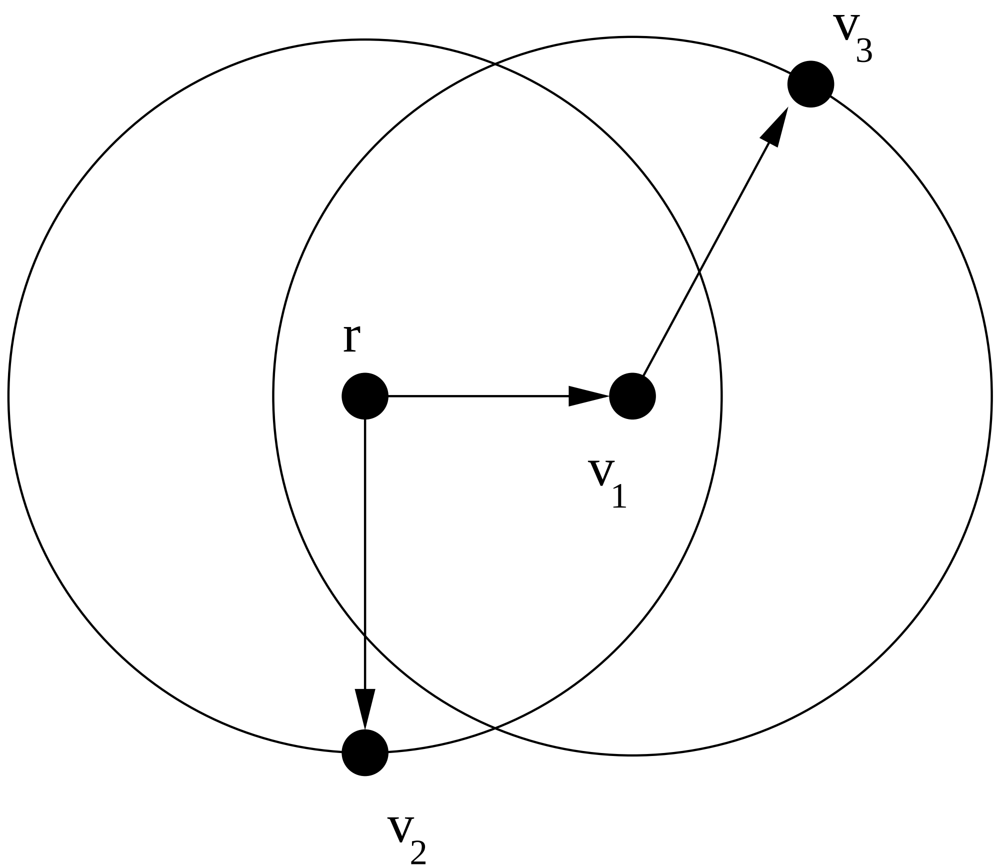

Figure 2: Node r can reach v_1 and v_2 with one single transmission. Node v_1 relays to v_3

Existing Algorithms for Broadcast

Best approximation ratio: $2 \log n$ (Calinescu et. al. ESA 2003.)

Interesting special cases:

1. power requirements symmetric ($c(u.v) = c(v, u)$) - same algorithm
2. Euclidean case: $c(u, v) = d(u, v)^\kappa$, where $d(u, v)$ the Euclidean distance between u and v and κ is the signal attenuation exponent, which is assumed to be in between 2 and 5 and is the same for all pairs of nodes - Minimum Spanning Tree algorithm has approximation ratio 6 (Ambuehl, ICALP 2005)
3. Line case: Euclidean case when all nodes lie on a single line - has polynomial-time algorithms

Hitch-hiking advantage

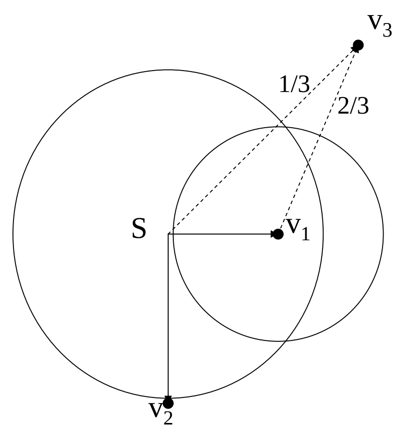

Figure 3: An illustration of the advantage of hitch-hiking. Node v_3, who is equiped with a maximal ratio combiner, receives 1/3 of the information from s and 2/3 from v_1.

Formal Definition

The input consists of a complete directed graph $G = (V, E)$ with power requirement function $c : E \to R^+$, and a source $s \in V$.

The output consists of a permutation $\tau =< v_1, v_2, \ldots, v_n >$ of V with $v_1 = s$ and power assignment $p(v)$ of every vertex v. For every $1 \leq i < j \leq n$, define $q(v_i v_j) = p(v_i)/c(v_i v_j)$. An output is feasible if for every $j > 1$ we have $\sum_{i=1}^{j-1} q(v_i v_j) \geq 1$. The objective is to minimize $\sum_{i=1}^{n} p(v_i)$.

Note: This is an idealized version!

Finding the permutation is hard!

Note that once the permutation τ is given, the best values for $p(v_i)$ are given by solving a linear program with variable $p(v_i)$ for all i and $q(v_iv_j)$ for all $i < j$. We call this linear program LP_1:

$$\text{Minimize } \sum_{v \in V} p(v) \text{ subject to}$$

$$
\begin{aligned}
q(v_iv_j)c(v_iv_j) &\leq p(v_i) && \forall 1 \leq i < j \leq n && (1)\\
\textstyle\sum_{i=1}^{j-1} q(v_iv_j) &\geq 1 && \forall j \geq 2. && (2)\\
q(v_iv_j) &\geq 0 && \forall 1 \leq i < j \leq n && (3)\\
p(v_i) &\geq 0 && \forall 1 \leq i \leq n && (4)
\end{aligned}
$$

Previous Results

- Maric and Yates, JSAC 2004: NP-Hardness and heuristics/simulation
- Manish Agarwal, Joon Ho Cho, Lixin Gao, and Jie Wu: INFOCOM 2004 - heuristics and simulation: up to 50% savings versus No-Hitch-hiking Broadcast

Our Results

- Theorem 1: $OPT_B \leq c \log^2 n \quad OPT_{HB}$
- Theorem 2: there are instances with $OPT_B \geq c' \log^2 n \quad OPT_{HB}$
- Such examples exist for also for Unicast!
- Line case: $OPT_B \leq 2 \frac{\frac{\pi^2}{6}}{2 - \frac{\pi^2}{6}} \; OPT_{HB}$

Proof of Theorem 1

The proof is based on a probabilistic argument. A new power function $\overline{p}(v)$ for $v \in V$ is constructed. For $1 \leq i < j \leq n$, edges $v_i v_j$ are said to be *selected* when $\overline{p}(v_i) \geq c(v_i v_j)$, or in other words the packet sent by v_i would be decoded completely by v_j. For $j > 1$, we call the vertex v_j *covered* if there some $i < j$ with $v_i v_j$ selected. One can easily check that if every vertex is covered, then we have a feasible solution to the Min-Energy Broadcast problem.

We assign power $\overline{p}_i = \overline{p}(v_i)$ as follows: for each i, independently pick $x_i \in (0, 1)$.

1. If $x_i < \frac{1}{2n}$, set $\overline{p}_i = 2p_i n$.
2. If $x_i \geq \frac{1}{2n}$, set $\overline{p}_i = \frac{2p_i}{x_i}$.

Proof of Theorem 1 - Continued

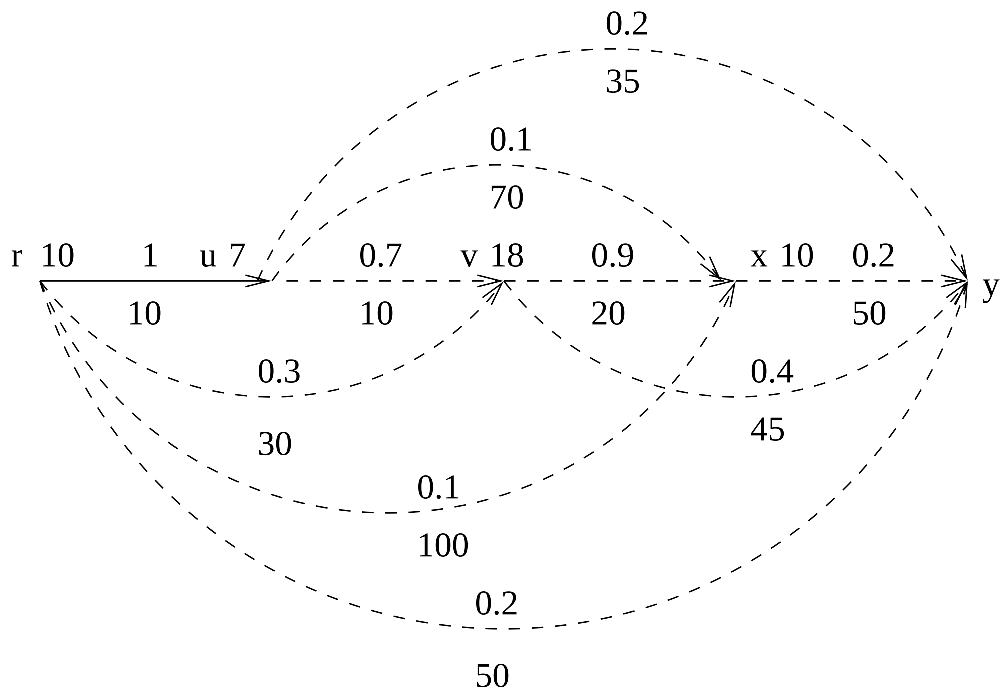

Figure 4: A solution with hitch-hiking

Proof of Theorem 1 - Continued

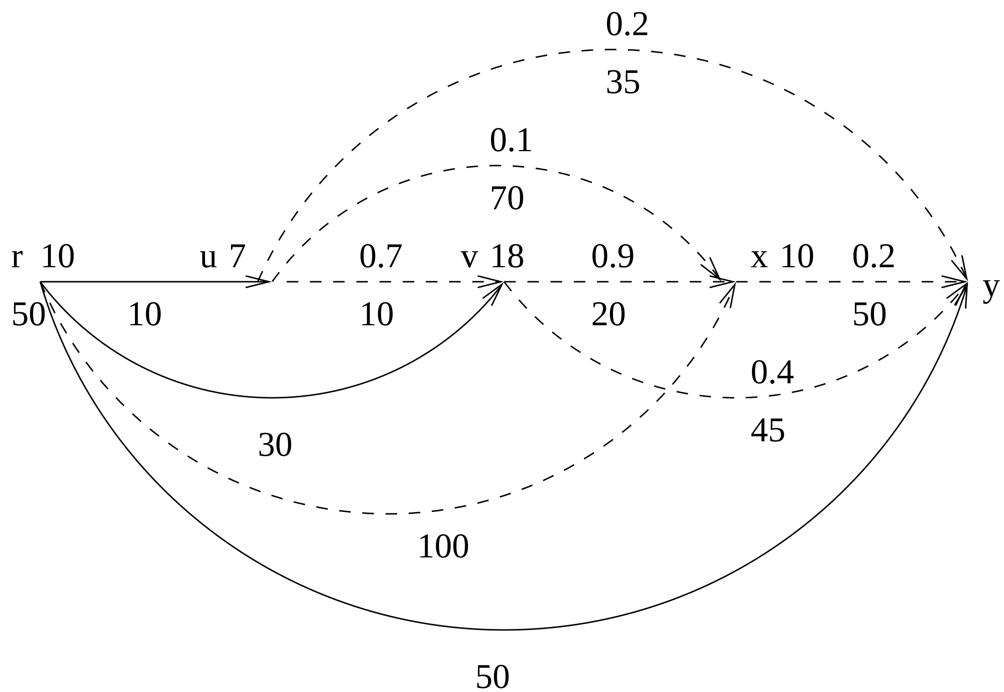

Figure 5: "Randomly" we pick $x_r = 0.4$. Then $\overline{p}_r = \frac{2p_r}{0.4} = 50$.

Proof of Theorem 1 - Continued

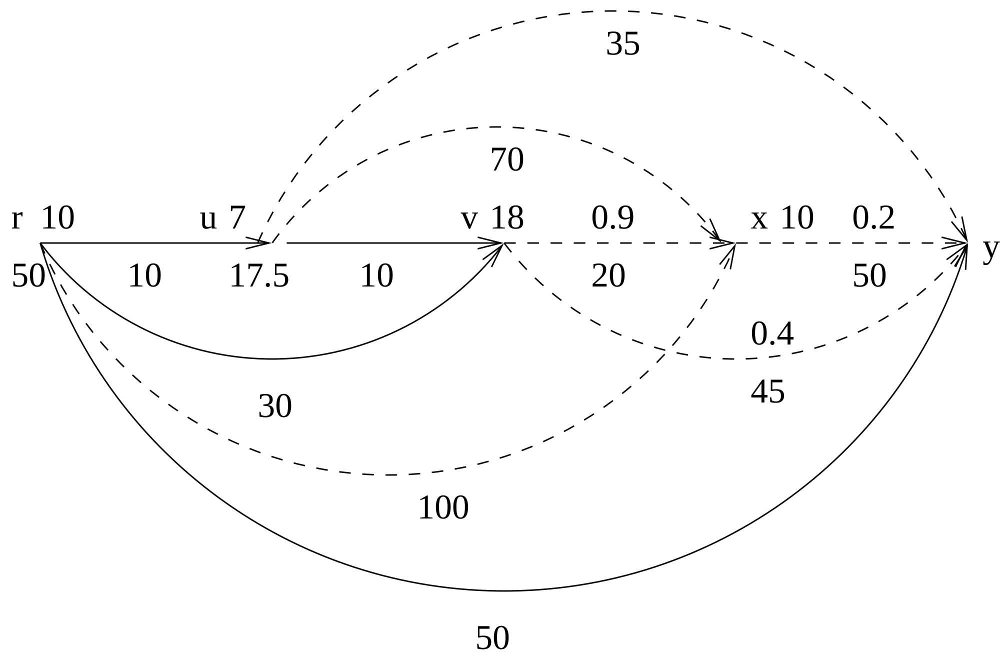

Figure 6: "Randomly" we pick $x_u = 0.8$. Then $\overline{p}_u = \frac{2p_u}{0.8} = 17.5$.

Proof of Theorem 1 - Continued

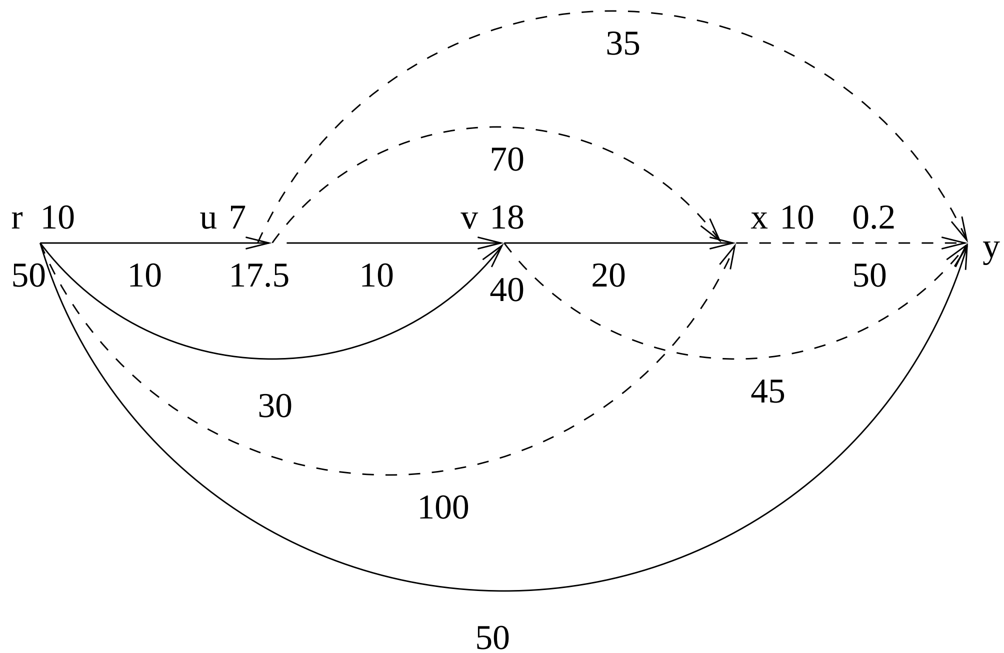

Figure 7: "Randomly" we pick $x_v = 0.9$. Then $\overline{p}_v = \frac{2p_v}{0.9} = 40$.

Proof of Theorem 1 - Continued

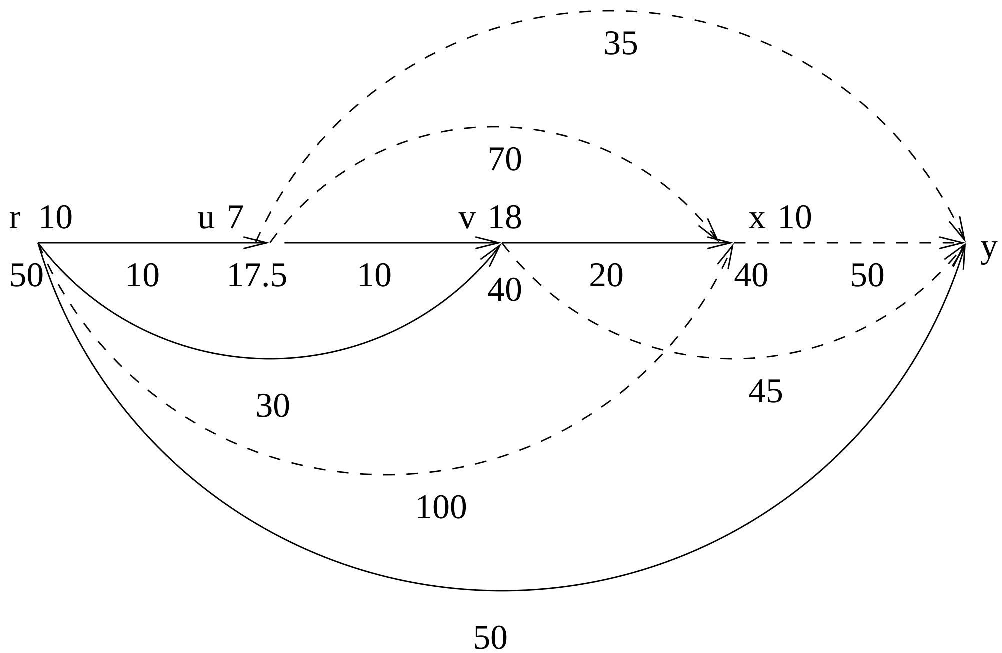

Figure 8: "Randomly" we pick $x_x = 0.5$. Then $\overline{p}_x = \frac{2p_x}{0.5} = 40$.

Proof of Theorem 1 - Continued

Then the expected value

$$E[\overline{p}_i] = \frac{1}{2n} \cdot 2p_i n + \int_{1/2n}^{1} \frac{2p_i}{x_i} = p_i + 2p_i(\ln 1 - \ln \frac{1}{2n}) = p_i(1 + 2\ln 2n),$$

and thus the total expected power of the random power assignment $\overline{p}$ is

$$E[\sum_{i=1}^{n} \overline{p}_i] = \sum_{i=1}^{n} E[\overline{p}_i] = (1 + 2\ln 2n) \sum_{i=1}^{n} p_i.$$

If nodes u and v satisfy $\frac{1}{2n} \leq q(uv) \leq \frac{1}{2}$, we have that the probability edge uv is selected is equal to

$$Pr[\overline{p}(u) \geq c_{uv}] \geq Pr[\overline{p}(u) \geq \frac{p(u)}{q(uv)}] = Pr[\frac{2p(u)}{x_u} \\ \geq \frac{p(u)}{q(uv)}] = Pr[x_u \leq 2q(uv)] = 2q(uv),$$

where we used the constraint $q(uv) \leq \frac{p(u)}{c_{uv}}$, the fact that $\frac{p(u)}{q(uv)} \leq 2np(u)$ and the fact that x_u is uniformly picked from the interval $(0, 1)$. Thus for every u with u before v in τ, the probability uv is selected is at least:

$$z_{uv} = \begin{cases} 1 \text{ if } q(uv) > \frac{1}{2} \\ 2q(uv) \text{ if } \frac{1}{2n} \leq q(uv) \leq \frac{1}{2} \\ 0 \text{ if } q(uv) < \frac{1}{2n} \end{cases}$$

Proof of Theorem 1 - Continued

From now on we follow the exposition from Vazirani's book closely in computing the probability that a node v is covered by a selected edge. Since $\sum_u q(uv) \geq 1$, and we have n nodes, $\sum_{u:q(uv)\geq\frac{1}{2n}} q(uv) \geq \frac{1}{2}$ and therefore $\sum_u z_{uv} \geq 1$. Using elementary calculus, it is easy to show that under this condition, the probability that v is covered by selected edges is minimized when each of the z_{uv} is $1/n$. Using the fact that each vertex u independently chooses $\overline{p}(u)$, we have:

$$Pr[v \text{ is covered by selected edges}] \geq 1 - \left(1 - \frac{1}{n}\right)^n \geq 1 - \frac{1}{e},$$

where e is the base of natural logarithms. Hence each node is covered with constant probability by the selected edges.

Proof of Theorem 1 - Continued

for every u, and among them use the largest value when computing $\overline{p}(u)$, where c is a constant such that

$$\left(\frac{1}{e}\right)^{c\log n} \leq \frac{1}{4n}.$$

Now,

$$Pr[v \text{ is not covered by selected edges}] \leq \left(\frac{1}{e}\right)^{c\log n} \leq \frac{1}{4n}.$$

Summing over all nodes $v \in V$, we get

$$Pr[\text{there is vertex } v \text{ not covered by selected edges}] \leq n \cdot \frac{1}{4n} \leq \frac{1}{4}.$$

Proof of Theorem 1 - Continued

We obtain $E[\sum_{i=1}^{n} \overline{p}_i] \leq c \log n(1 + 2\ln 2n) \sum_{i=1}^{n} p_i$. Applying Markov's Inequality we obtain

$$Pr[\sum_{i=1}^{n} \overline{p}_i \geq 4c \log n(1 + 2\ln 2n) \sum_{i=1}^{n} p_i] \leq \frac{1}{4}.$$

The probability of the union of the two undesirable events is $\leq 1/2$. Hence,

$$Pr[\sum_{i=1}^{n} \overline{p}_i \leq 4c \log n(1{+}2\ln 2n) \sum_{i=1}^{n} p_i \text{ and every } v \text{ is covered by selected edges}] \geq \frac{1}{2}.$$

Thus there exist a valid power assignment without hitch-hiking of total power at most $4c \log n(1 + 2\ln 2n) \sum_{i=1}^{n} p_i$, completing the proof of the theorem.

Idea of Theorem 2

There are instances with $OPT_B \geq c' \log^2 n \quad OPT_{HB}$

Construction from Vazirani's book: For a positive k, we get a bipartite graph H with the vertex set $V(H) = A \cup B$, A, B disjoint, and $n/2 = |A| = |B| = 2^k - 1$; every vertex of B has degree 2^{k-1}. Moreover, any subset $S \subseteq A$ "covering" B must have at least $k = \Theta(\log n)$ vertices. We reproduce this construction many times.

Idea of Theorem 2

This only gives instance with $OPT_B \geq c' \log n \quad OPT_{HB}$

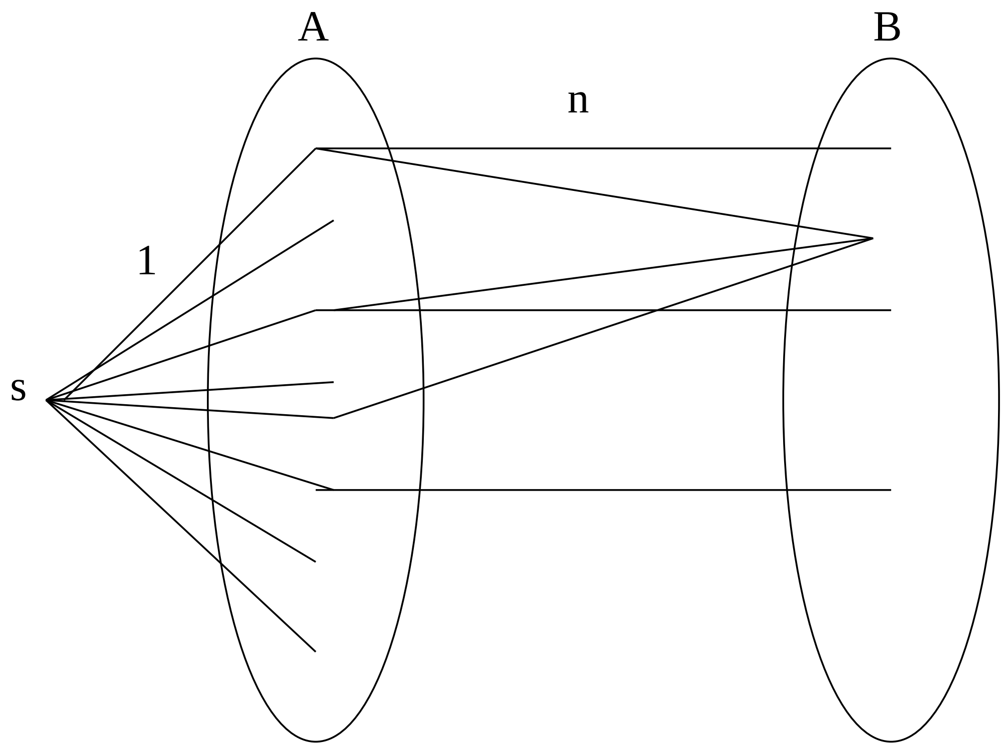

Figure 9: In Broadcast without Hitch-Hiking, $\Theta(\log n)$ nodes of A must have power n; with Hitch-Hiking every node of A has power 4 as every node of B has degree $n/4$.

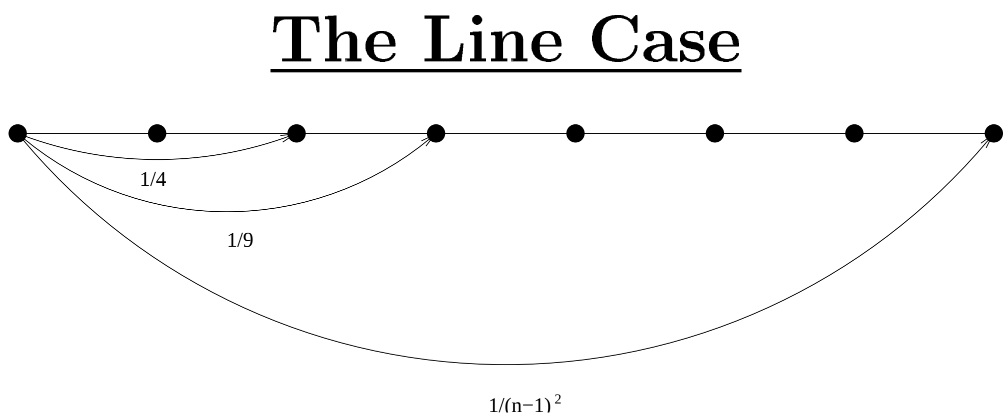

Figure 10: We believe this is gives the largest OPT_B/OPT_{BH} in the Line case - the ratio would be $1 + 1/4 + 1/9 + \ldots + 1/n^2 \leq \pi^2/6$.

Approximation Algorithms

The known algorithms for Broadcast achieve for Broadcast with Hitch-Hiking:

1. approximation ratio $O(\log^3 n)$ with arbitrary asymetric power requirements
2. approximation ratio $O(\log^2 n)$ in the Euclidean case;
3. constant approximation ratio in the Line case

Also, for Unicast $O(\log^2 n)$ approximation ratio.

Conclusions and Open Problems

We achieved:

1. Tight, up to a constant, bounds of OPT_B/OPT_{HB} in the case of arbitrary power requirements and in the line case

2. Approximation algorithms for Broadcast with Hitch-Hiking

Left Open:

1. A realistic model of Hitch-Hiking

2. Ratio of OPT_B/OPT_{HB} in the Euclidean case

3. Polynomial-time algorithm for Broadcast with Hitch-Hiking in the Line case

4. Better Approximation Algorithms for Broadcast with Hitch-Hiking