Connected Dominating Sets in Wireless Networks

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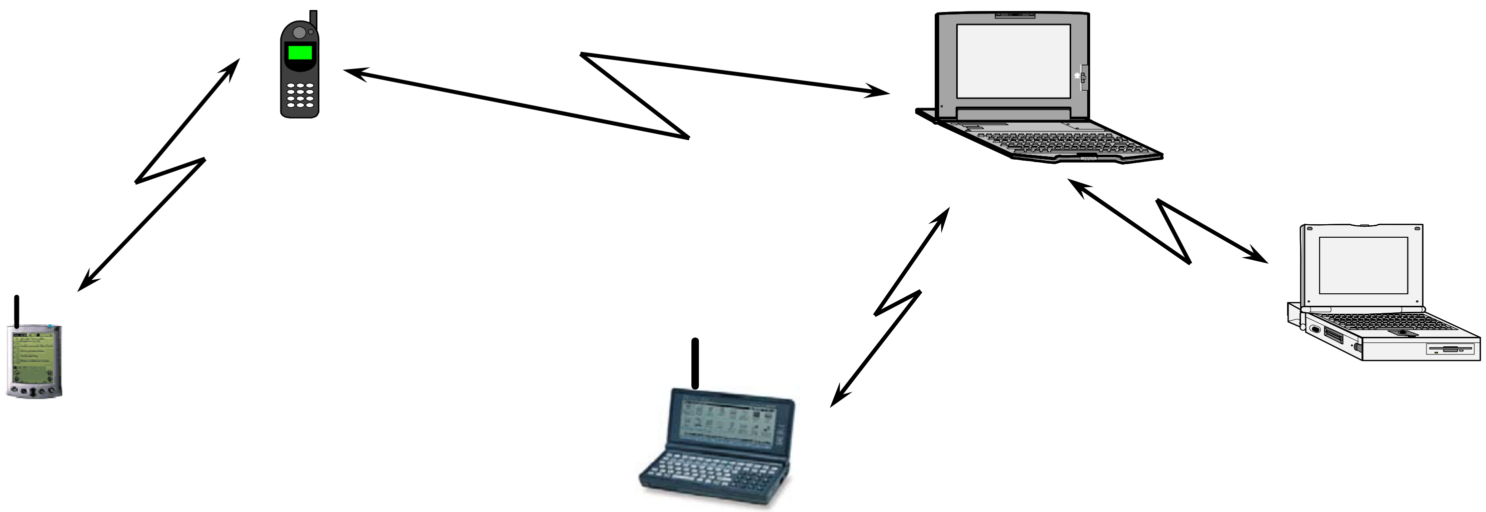
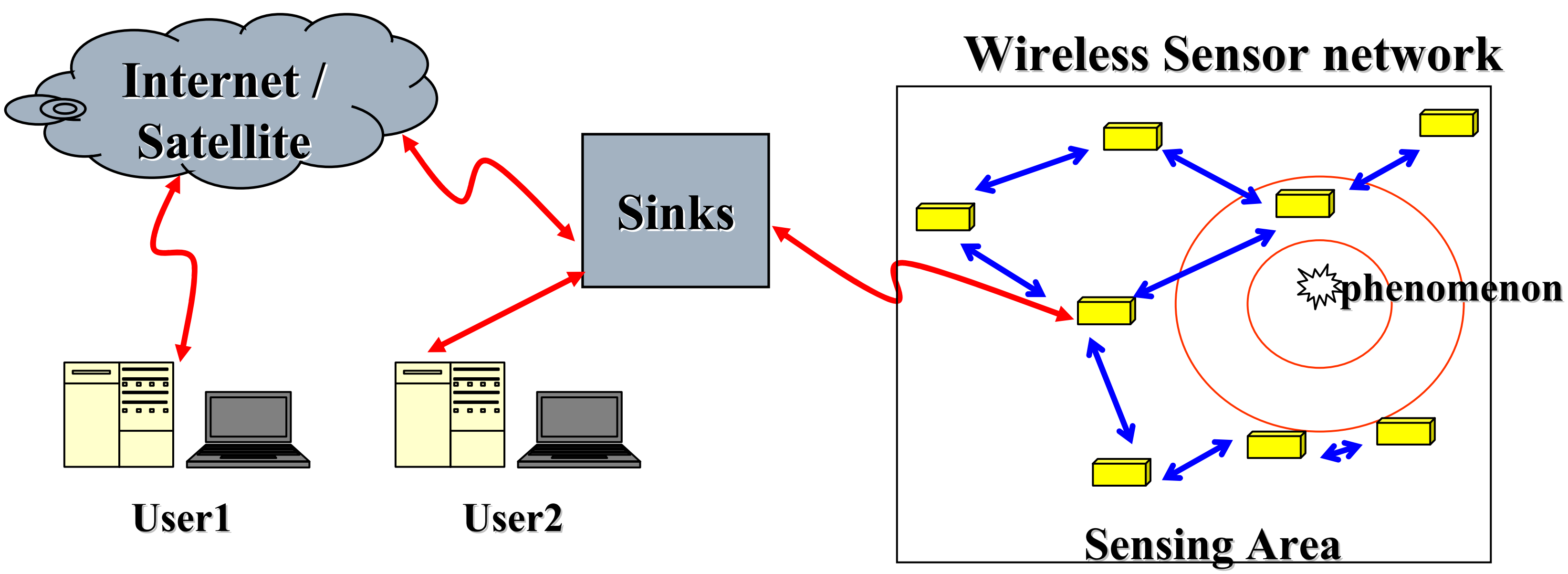
Wireless Ad Hoc Networks

- A collection of mobile nodes
- Dynamically form a temporary network



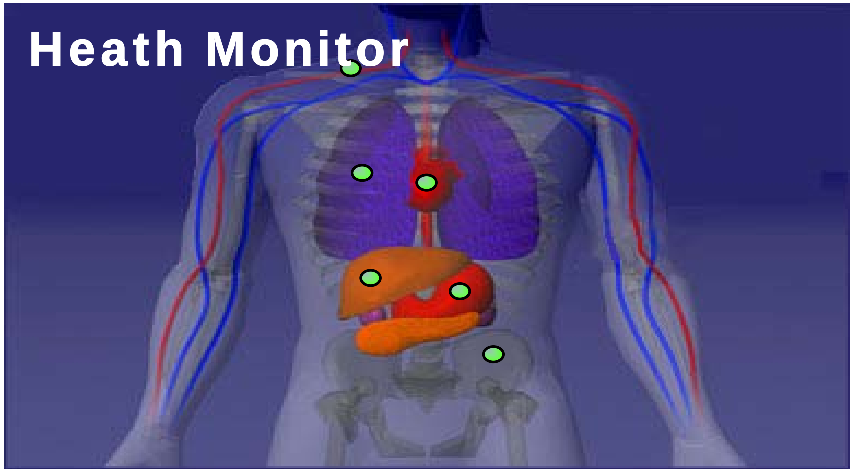
Wireless Sensor Networks (WSN)

- Consists of a large number of sensor nodes
- **Main Tasks**: **collaborate** to sense, collect, and process the raw data of the **phenomenon** and transmit the processed data to **sinks**



Applications

- Military applications
- Environmental applications
- Health applications
- Other commercial applications

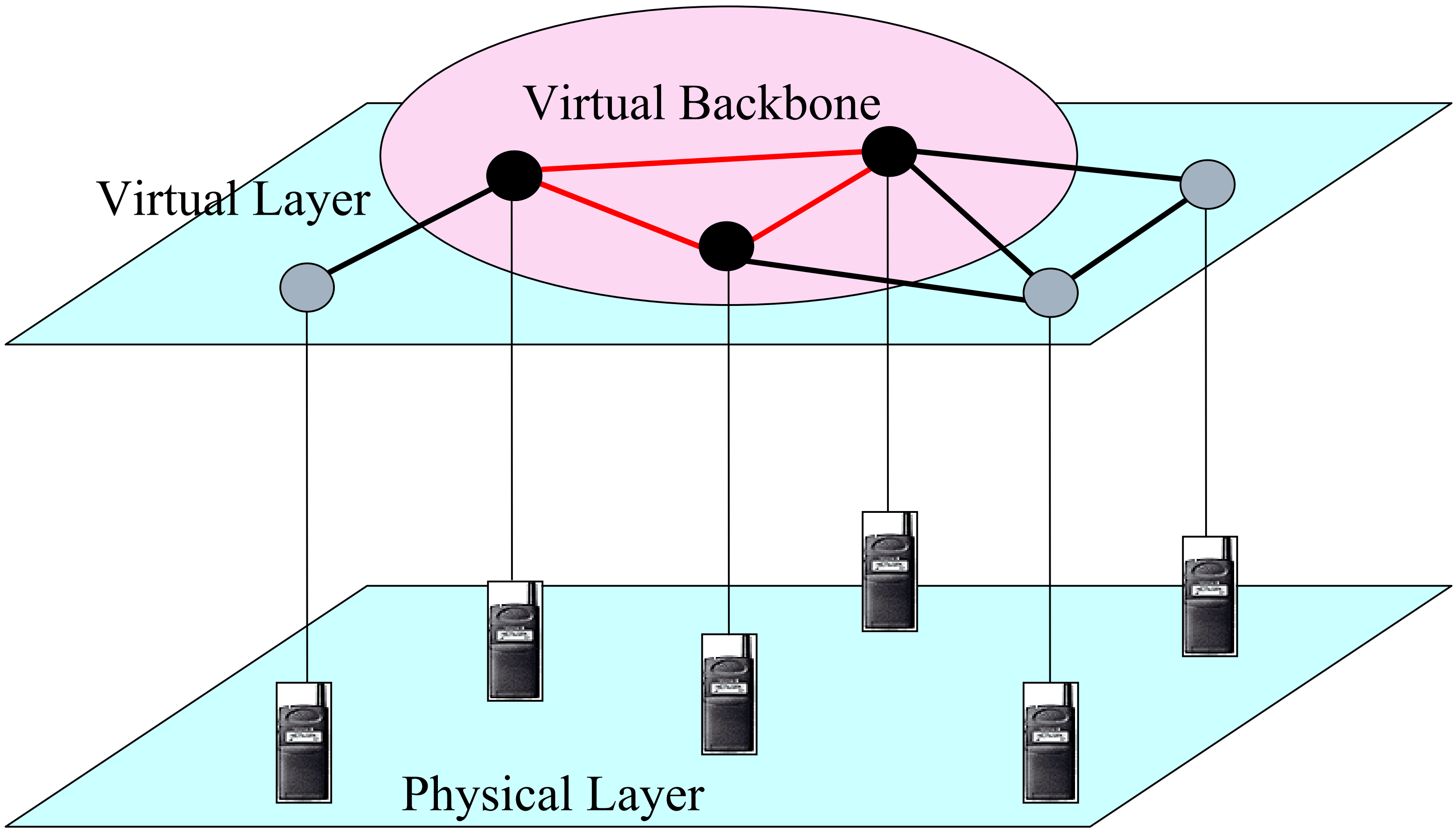


Characteristics of Wireless Ad Hoc Networks

- Dynamic topology – no predefined or fixed infrastructure
- Multi-hop routing – each node is a router
- Limited resources – battery power, CPU, storage, and bandwidth

Routing decision is challenging!

Virtual Backbone

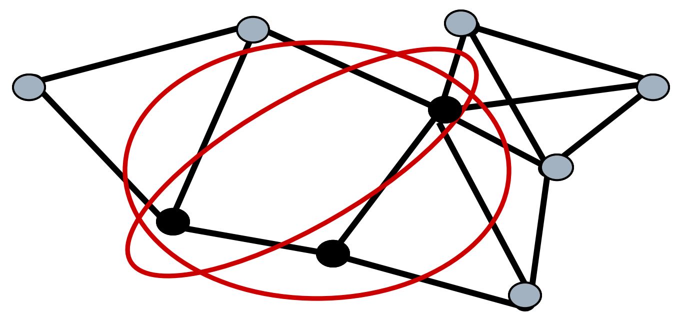


Virtual Backbone Features

- Minimize the virtual backbone nodes
- All virtual backbone nodes are connected
- Each node is either in or adjacent to the backbone
- Approximated by Minimum Connected Dominating Set (MCDS)

Definitions

- Given a graph $\boldsymbol{G}=(\boldsymbol{V},\boldsymbol{E})$ and a subset $\boldsymbol{C} \subseteq \boldsymbol{V}$. $\boldsymbol{C}$ is:
  - Dominating Set (DS): for any $\boldsymbol{v} \in \boldsymbol{V}$, $\boldsymbol{v} \in \boldsymbol{C}$ or adjacent to some $\boldsymbol{u} \in \boldsymbol{C}$
  - Connected Dominating Set (CDS): $\boldsymbol{C}$ is a DS and an induced graph of $\boldsymbol{C}$ is connected
  - Minimum Connected Dominating Set (MCDS): C is a CDS and has the smallest size



Approximation Algorithms

- An algorithm that returns near-optimal solutions in polynomial time
- Performance Ratio (PR):
  - Minimization problem: $|C|/|C^*|$ where:
    - C is a near-optimal **MCDS**
    - C^* is the optimal **MCDS**
  - Smaller PR, better algorithm

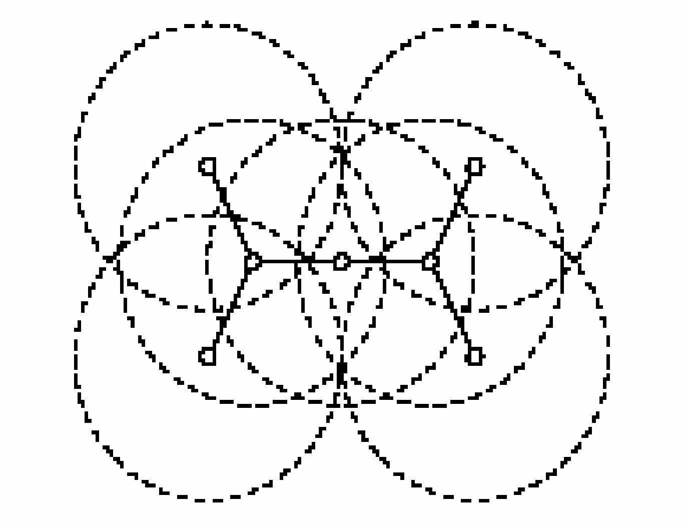
Homogeneous Networks

What if all nodes have the same transmission ranges?

Can we design a constant approximation algorithm?

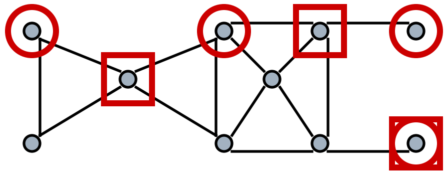
Unit Disk Graphs (UDG)

- UDG: is an intersection graphs of circles of unit radius in the plane



- Lemma 1: Each node in a UDG has at most 5 independent neighbors

Maximal Independent Set (MIS)

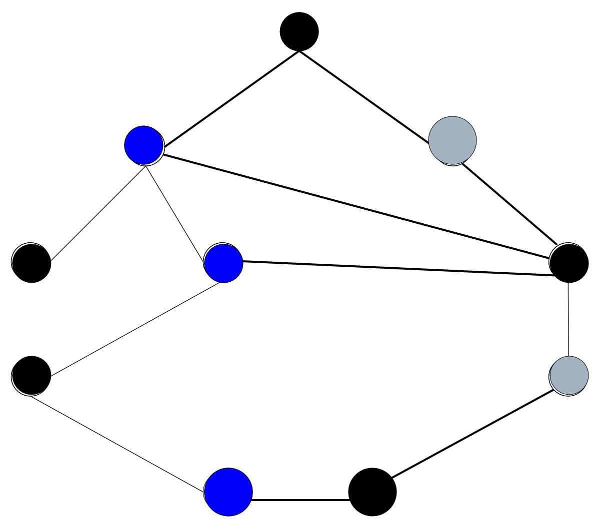


Maximal Independent Set (MIS) is a maximal set of pairwise non-adjacent nodes.

MIS ➡ DS

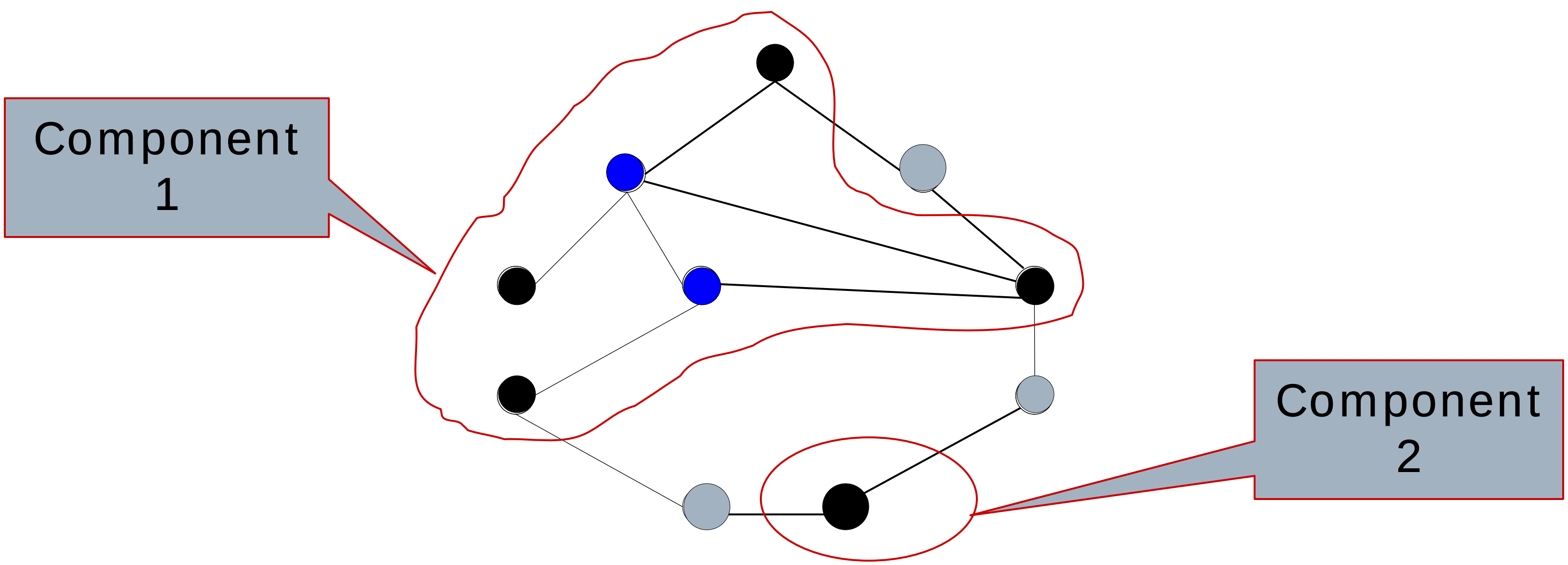
Algorithm 1 - Overview

- Phase 1: Construct an MIS such that:
  - Lemma 2: Any pair of complementary subsets of the MIS separate by exactly two hops
- Phase 2: Connect MIS ➡ CDS



Algorithm 1 – Phase 2

- **Goal:** Connect an MIS by adding the minimum number of blue nodes where:
  - Blue Nodes: Nodes connecting black nodes
- Black-blue component: a connected component of the sub-graph induced only by black and blue nodes



Algorithm 1 – Phase 2 (cont)

Input: An MIS:

- All nodes in MIS are black
- Others are grey

for *i*=5, 4, 3, 2 do

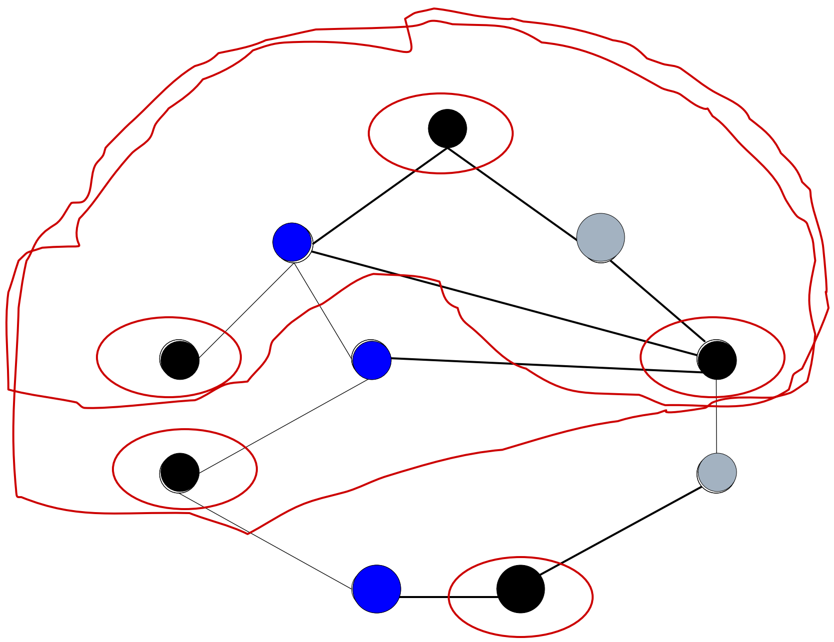
while there exists a grey node adjacent to at least *i* black-blue components ***do***

change its color from grey to blue

re-construct the black-blue components

return all black and blue nodes

Algorithm 1 – An Example



Algorithm 1 - Analysis

- Theorem 1: Algorithm 1 has a performance ratio of **5.8 + *ln* 4 < 8**
  - Lemma 3: **|MIS| ≤ 3.8|C*| + 1.2**
  - Lemma 4: **#Blue Nodes ≤ (2 + ln4)|C*|**

Fault Tolerance

What if a virtual backbone node is dead?

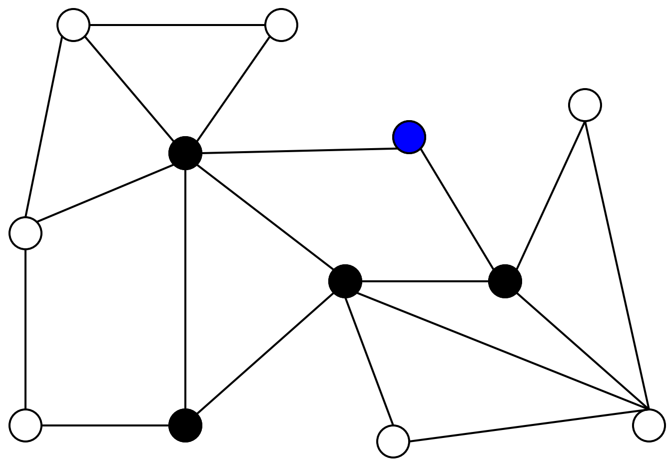
What if a link in the virtual backbone is broken?

2-CDS

- **Problem Definition:**
  - Given a UDG ***G=(V,E)***
  - Find a CDS ***C*** satisfying:
    - |***C***| is minimum
    - For any pair of nodes in ***C***, there exists 2-disjoint paths

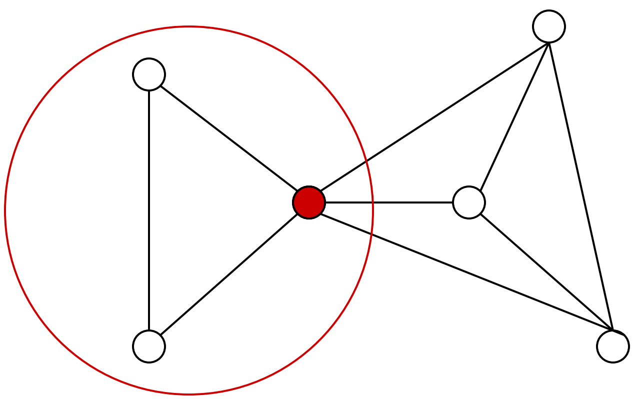
Algorithm 2 - Overview

- Phase 1: Construct a CDS ***C***
- Phase 2: Augment ***C*** to obtain a 2-CDS



Algorithm 2 – Phase 2

- Cut-vertex: ***x*** is a cut-vertex if ***G***-{***x***} is disconnected
- Block: a maximal subgraph of ***G*** without cut-vertices



- Leaf Block: a maximal subgraph of ***G*** with exactly 1 cut-vertex

Algorithm 2 – Phase 2 (cont)

while ***C*** has more than 1 blocks ***do***

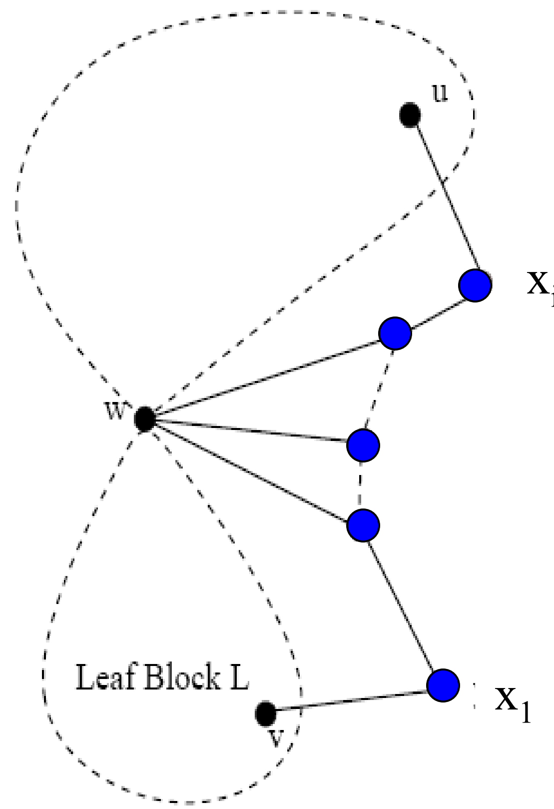
L = Leaf Block

Find a shortest path $P = ux_iv$ where $v \in L$, v is not a cut-vertex, $u \in C \setminus L$, and $x_i \in V \setminus C$

color x_i blue

end while

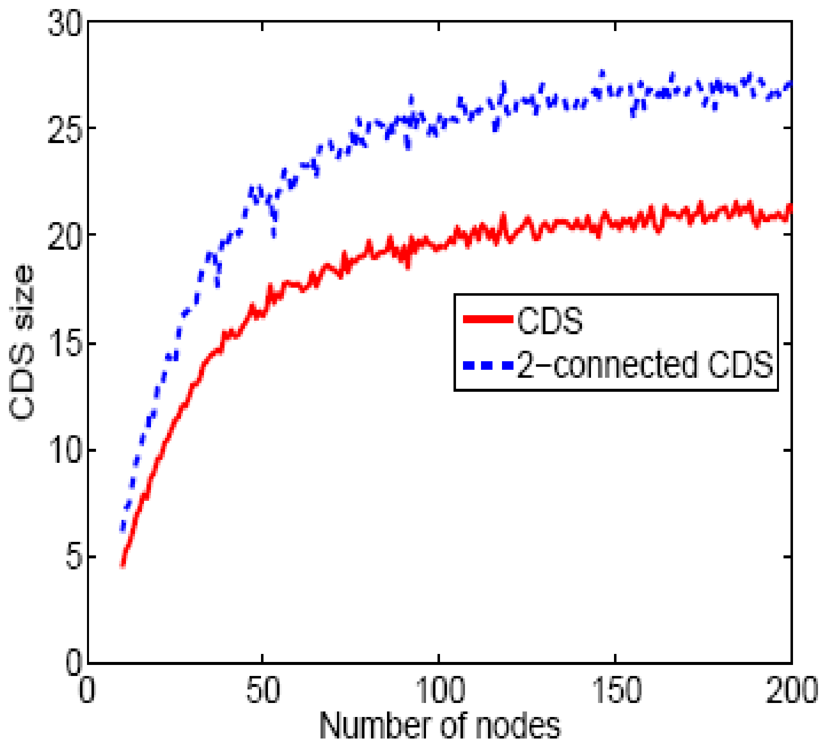
return all black and blue nodes



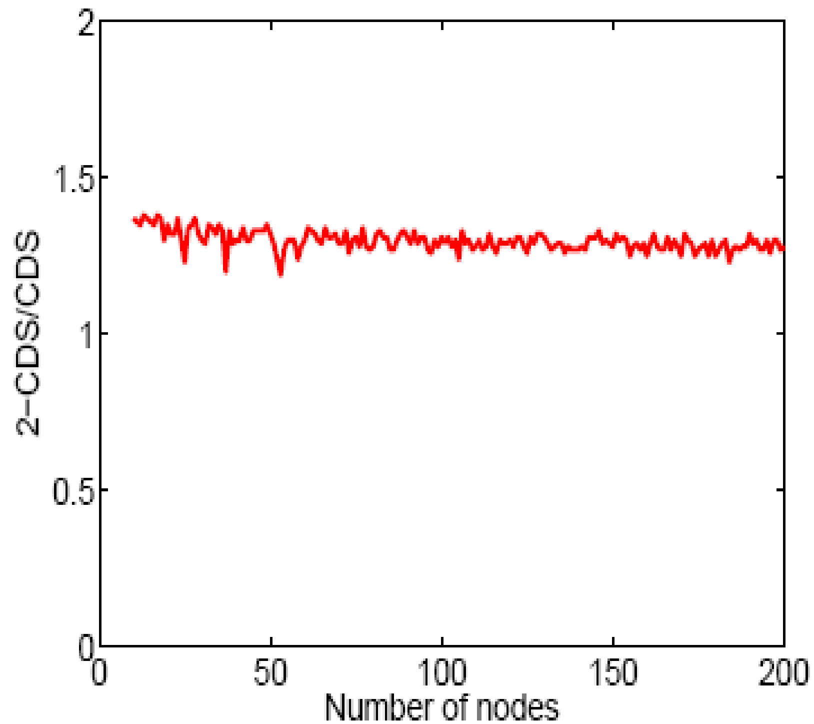
Theoretical Analysis

- Theorem 2: Algorithm 2 has a constant performance ratio of 62

Simulation Experiments



- Randomly deployed nodes into 1000 x 1000 m^2 region
- Transmission range = 200 m
- Each setting, ran 1000 times



Algorithm 2 improves the fault tolerance of virtual backbone with only marginal extra overhead

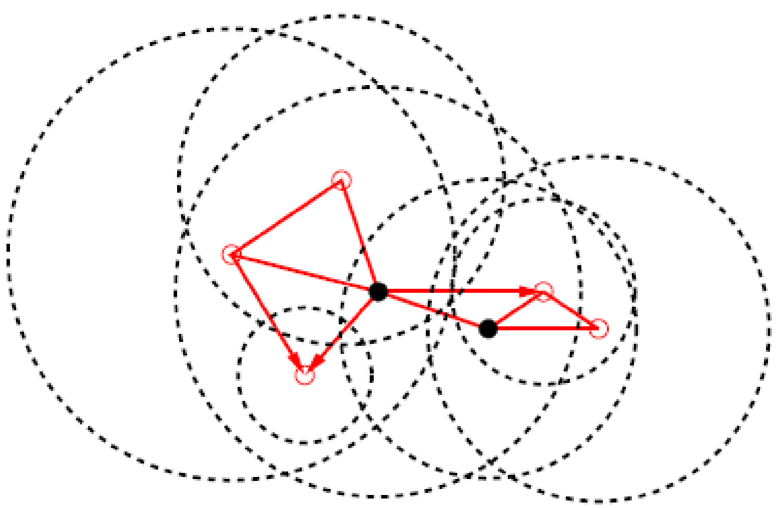
More Challenging

What if networks have unidirectional links and different transmission ranges?

Can we design a constant approximation algorithm?

Heterogeneous Networks

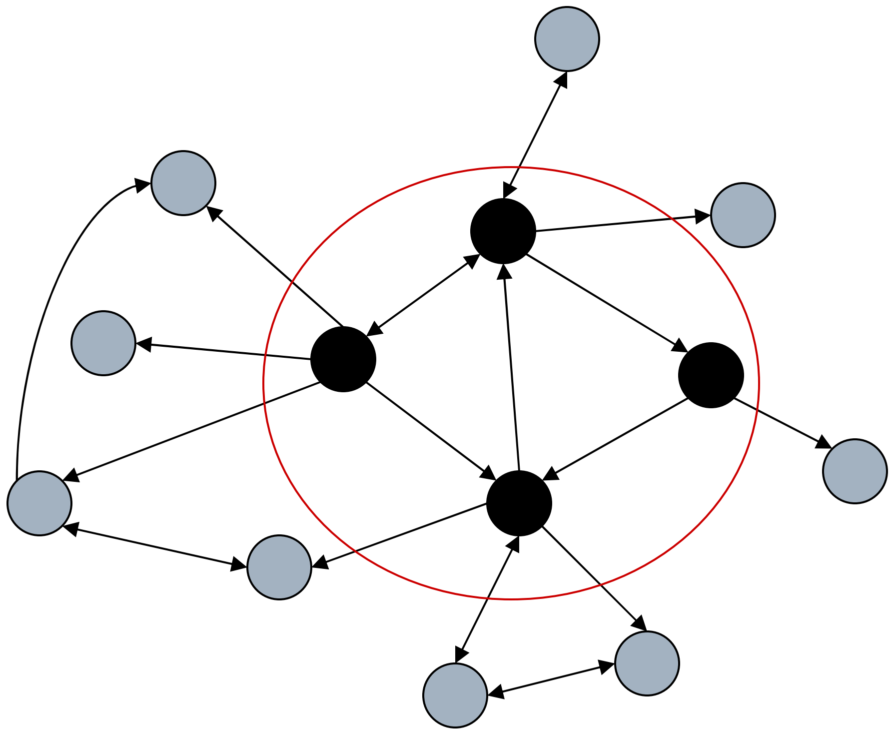
- Model networks as Disk Graphs:
  - Each node has a transmission range in $[r_{min}, r_{max}]$
  - A directed edge from u to v iff $d(u,v) \leq r_u$
- Bidirectional Links and Unidirectional Links



Unidirectional Links

- Directed graph $G = (V, E)$
- Strongly Connected Dominating Set (SCDS):
  - Given a directed graph $G = (V, E)$
  - Find a subset $C \subseteq V$ such that:
    - $\forall\, v \in V$, $v \in C$ or there exists a node $u \in C$ such that $uv \in E$
    - The subgraph induced by C is strongly connected, i.e, there exists a directed path for any pair of nodes in C

SCDS – An Example



Greedy Algorithm 3 - Overview

- Phase 1: Construct a DS ***D***
- Phase 2: Connects all nodes in ***D*** to form a SCDS ***C***

Algorithm 3 – Phase 1

while there exists a white node ***do***

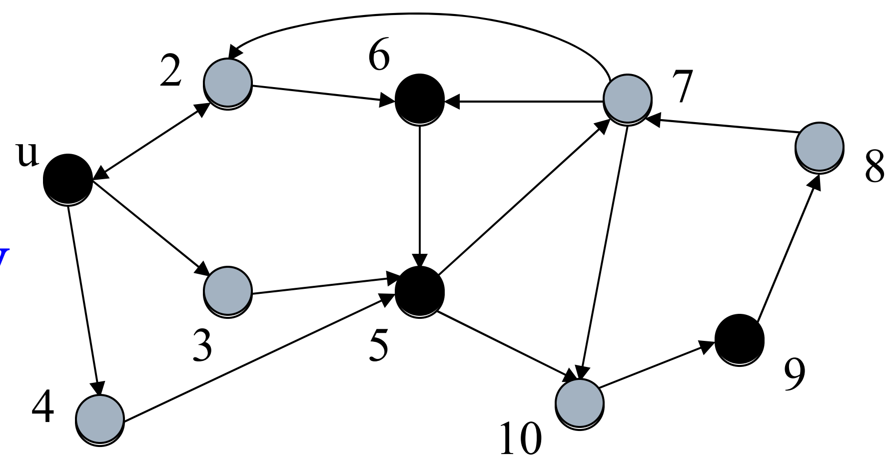
select a white node ***u*** with the biggest transmission range

color ***u*** black

color all $N^+(u)$ grey

end while

return all black nodes



Algorithm 3 – Phase 2

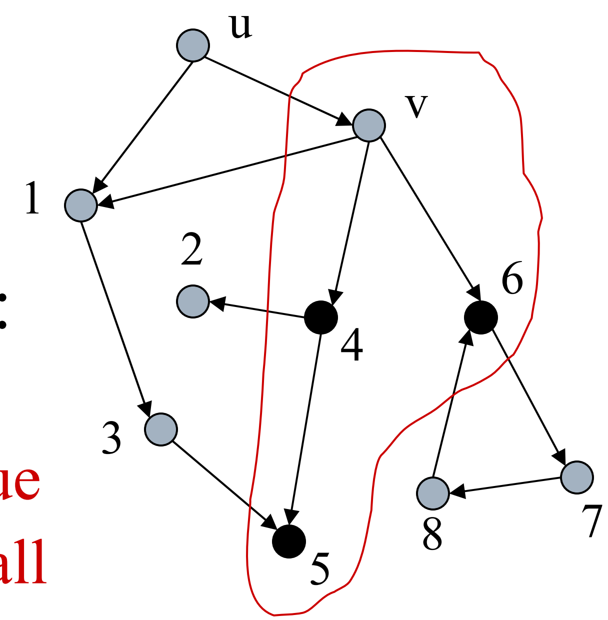
- Goal: Connect a DS D by adding the minimum number of blue nodes
  - Let $u \in D$ s.t. u has the largest transmission range
  - Build a Minimum nodes Directed Tree (MDT) T_1 rooted at u s.t. there is a directed path from u to all other nodes in D
  - Construct G' from G by reversing the directed edges
  - Build a MDT T_2 rooted at u
  - All nodes in the union of T_1 and T_2 form a SCDS C

Minimum nodes Directed Tree (MDT)

- Given a directed graph $\boldsymbol{G = (V, E)}$, a subset $\boldsymbol{D}$ of $\boldsymbol{V}$, and a node $\boldsymbol{u}$
- Find a tree $\boldsymbol{T}$ rooted at $\boldsymbol{u}$ such that:
  - There exists a directed path from $\boldsymbol{u}$ to all nodes in $\boldsymbol{D}$
  - The total number of nodes in $\boldsymbol{T}$ but not in $\boldsymbol{D}$, called blue nodes, is minimum

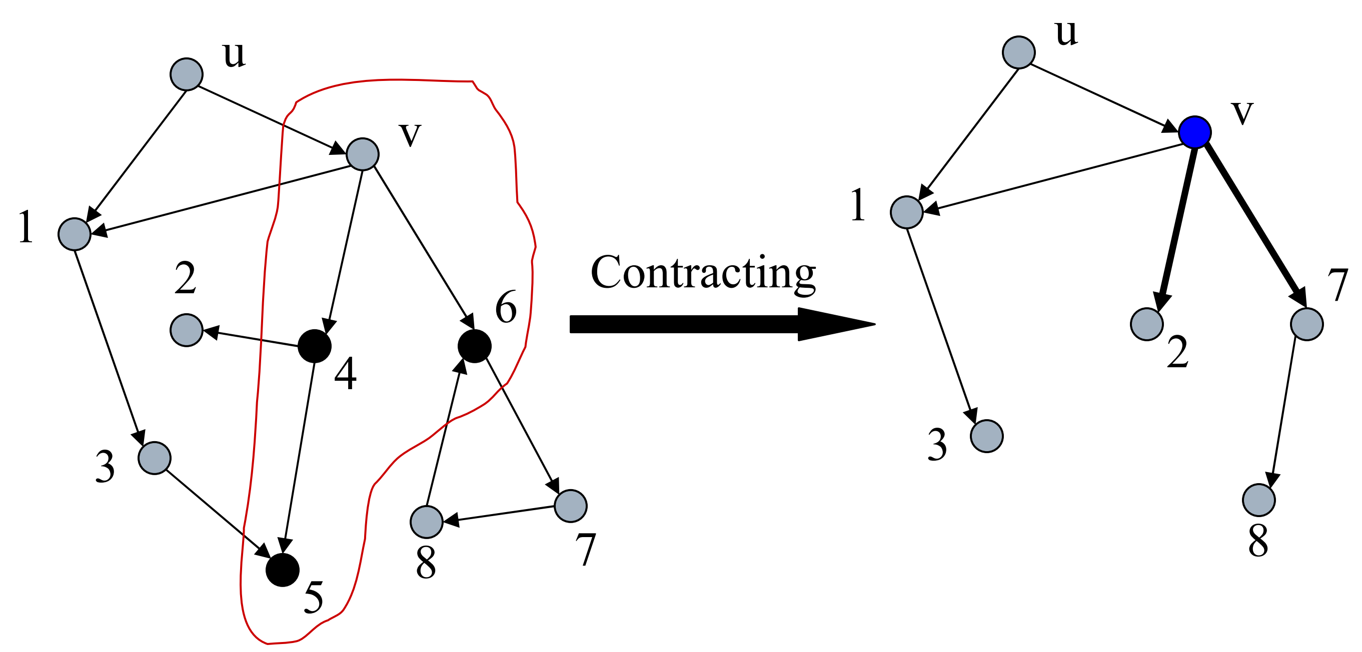
An MDT Algorithm

- Denote:
  - All nodes in ***D*** are black
  - All nodes in ***T*** \ ***D*** are blue
  - Other nodes are grey
- ***v***-spider: A directed tree having:
  - one grey node ***v*** as a root
  - other nodes are either black or blue
  - there is a directed path from ***v*** to all other nodes in the spider



An MDT Algorithm (cont)

- Contracting Operation:
  - Add a directed edge from v to all grey nodes that are outgoing neighbors of blue and black nodes in ***v***-spider
  - Delete all black and blue nodes and their edges
  - Color ***v*** blue



An MDT Algorithm - Description

while no directed paths from ***u*** to ***D*** in ***T*** ***do***

Find a ***v***-spider that has the most number of blue and black nodes

Contract this v-spider

Construct T from the set of black and blue nodes

end while

Algorithm 3 - Analysis

- Theorem 3: The size of any DS is upper bounded by:

$$2.4(k+\frac{1}{2})^2 |C^*| + 3.7(k+\frac{1}{2})^2 \text{ where } k = r_{\max} / r_{\min}$$

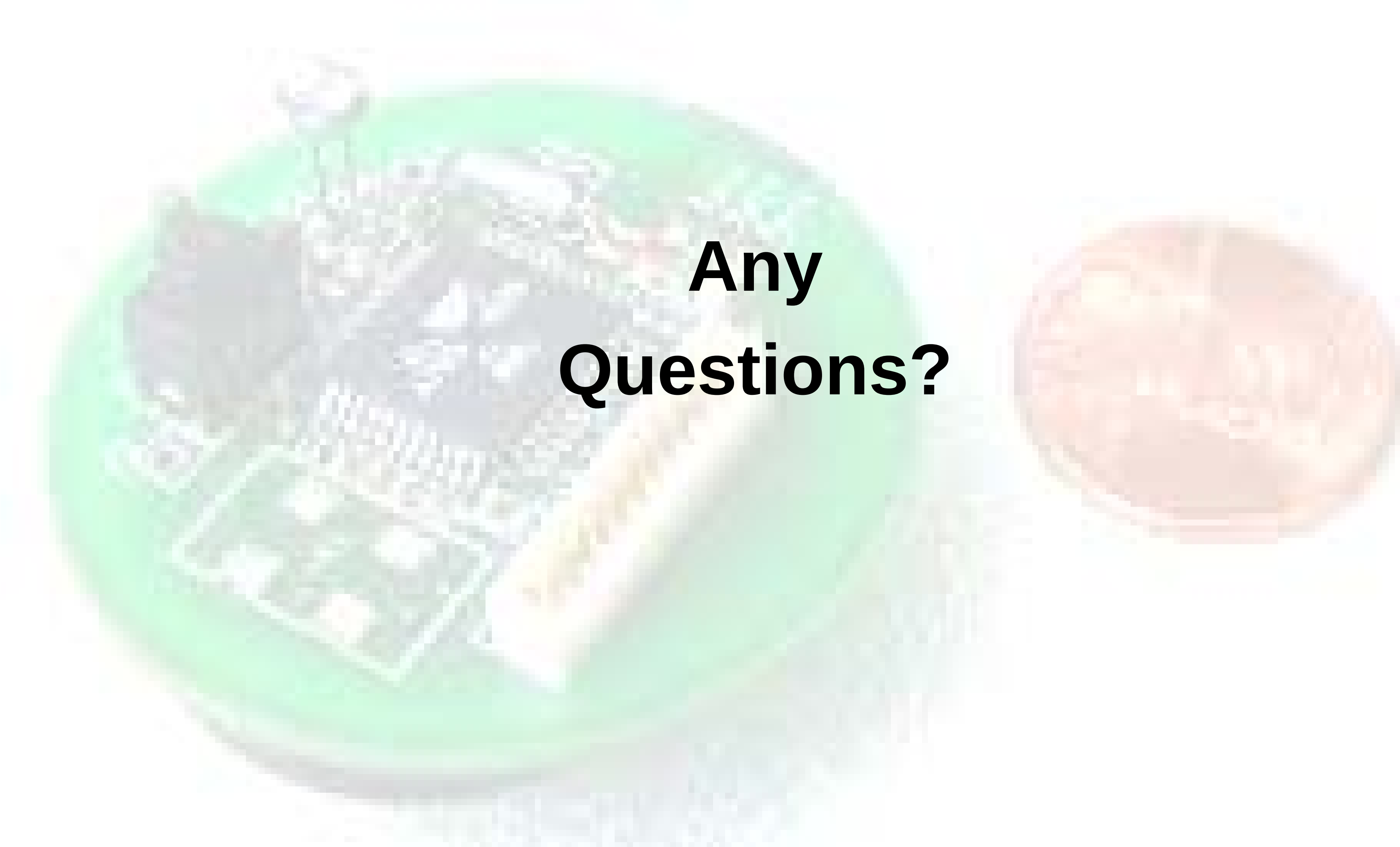
- Theorem 4: Algorithm 4 has a performance ratio of

$$2.4(k+\frac{1}{2})^2 + 4 + 4\ln(2k-1)$$

More Work

- k-connected m-dominating set
  - In the presentation: 2-connected 1-dominating set
- k-connected m-dominating set in heterogeneous networks

Thank You!

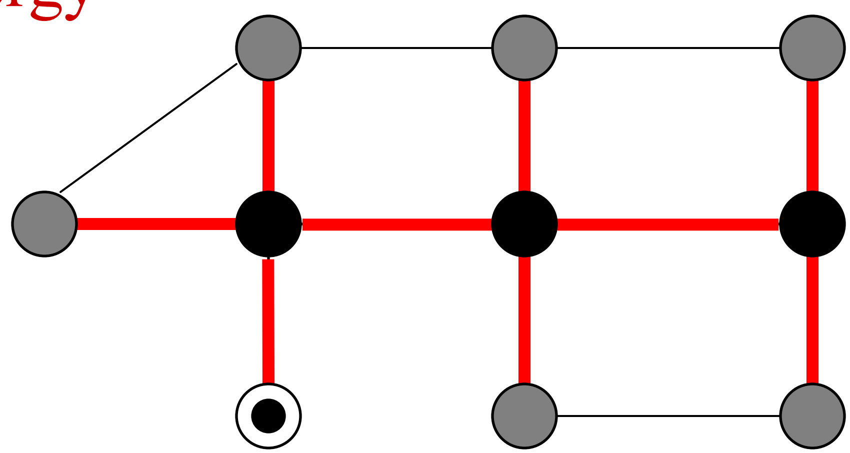


Any Questions?

Benefits of Virtual Backbone

Broadcast

- Only a subset of nodes (virtual backbone nodes) relay messages:
  - Reduce communication cost
  - Reduce redundant traffic
  - Conserve energy



Benefits of Virtual Backbone (cont)

Unicast

- Only a subset of nodes maintain routing tables
- Routing information localized
  - Save storage space