Maximum Lifetime of Sensor Networks with Adjustable Sensing Range

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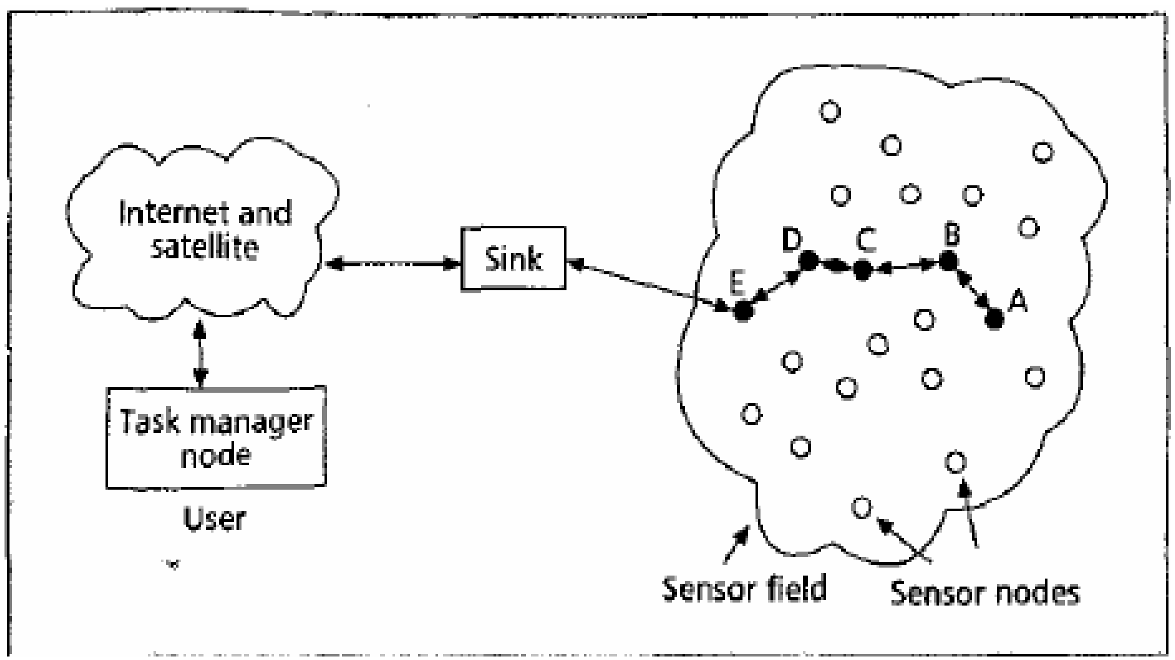
Outline

- Background
- Problem Statement and LP formulation
- The approximation algorithm
- Greedy solution to the dual problem
- Experimental Evaluation
- Discussion
- Conclusion

Introduction

- Sensor networks
- Major constraints – energy, computation, bandwidth



Introduction

- High node density implies that only a subset of nodes need to be active.
- *Target coverage problem* – A set of targets that need to be covered.
- *Idea:* Pick a set of active sensors as a number of set covers C_1, C_2, ..., C_m and use these one by one
- *Question:* How long? Need to assign a *time* to each cover. Pairs (C_m, t_m)

Adjustable range model

- Now lets make things more interesting…
- *Adjustable range* – Each sensor can vary its range from 0 (off) to *MAXDIST*
- So in addition to picking the sensors s_i that participate in (C_m, t_m) we need to associate a range r_i with each s_i
- Makes the problem more interesting because as range increases, target coverage increases but so does energy

Contributions

- Problem studied first by *Cardei et al [4]*
- We propose a different LP formulation
- Give a *provably* good heuristic
- Can handle non-uniform battery at each sensor
- Smooth sensing range model in place of discrete range model
- Initial results show 4 *x* improvement

Related work

- Cardei et al. [4]
- Maximize number of subsets – limit k

- c_k, boolean variable, for $k = 1..K$; $c_k = 1$ if this subset is a set cover, otherwise $c_k = 0$.
- x_{ikp}, boolean variable, for $i = 1..N$, $k = 1..K$, $p = 1..P$; $x_{ikp} = 1$ if sensor i with range r_p is in cover k, otherwise $x_{ikp} = 0$.

i: i^{th} sensor, when used as index
j: j^{th} target, when used as index
p: p^{th} sensing range, when used as index
k: k^{th} cover, when used as index

Maximize $c_1 + ... + c_K$
subject to

$$\sum_{k=1}^{K} \left(\sum_{p=1}^{P} x_{ikp} e_p\right) \leq E \quad \text{for all } i = 1..N$$
$$\sum_{p=1}^{P} x_{ikp} \leq c_k \quad \text{for all } i = 1..N,\ k = 1..K$$
$$\sum_{i=1}^{N} \left(\sum_{p=1}^{P} x_{ikp} * a_{ipj}\right) \geq c_k \quad \text{for all } k = 1..K,\ j = 1..M$$
$$x_{ikp} \in \{0, 1\} \text{ and } c_k \in \{0, 1\}$$

Sensor Network Lifetime Problem (SNLP) with range assignment

- Given a monitored region R, a set of sensors s_1, s_2, .. s_m and a set of targets $i_1, i_2, \ldots i_n$, and energy supply b_i for each sensor, find a monitoring schedule $(C_1,t_1), \ldots, (C_k,t_k)$ *and* a range assignment for each sensor in a set C_i such that:

(1) $t_1+\ldots+t_k$ is maximized,

(2) each set cover monitors all targets $i_1,\ldots,i_n$ and,

(3) each sensor s_i does not appear in the sets $C_1,..C_k$ for a time more than b_i

LP formulation

$$\textit{Maximize:} \sum_{j=1}^{m} t_j$$

$$\textit{Subject to} \quad \sum_{j=1}^{m} C_{ij}\, t_j \leq b_i \qquad (1)$$

where,

b_i is the battery for sensor *i*,

Rows i, i=1, …, n represent each sensor,

Columns j, j=1, …, m represent each sensor cover,

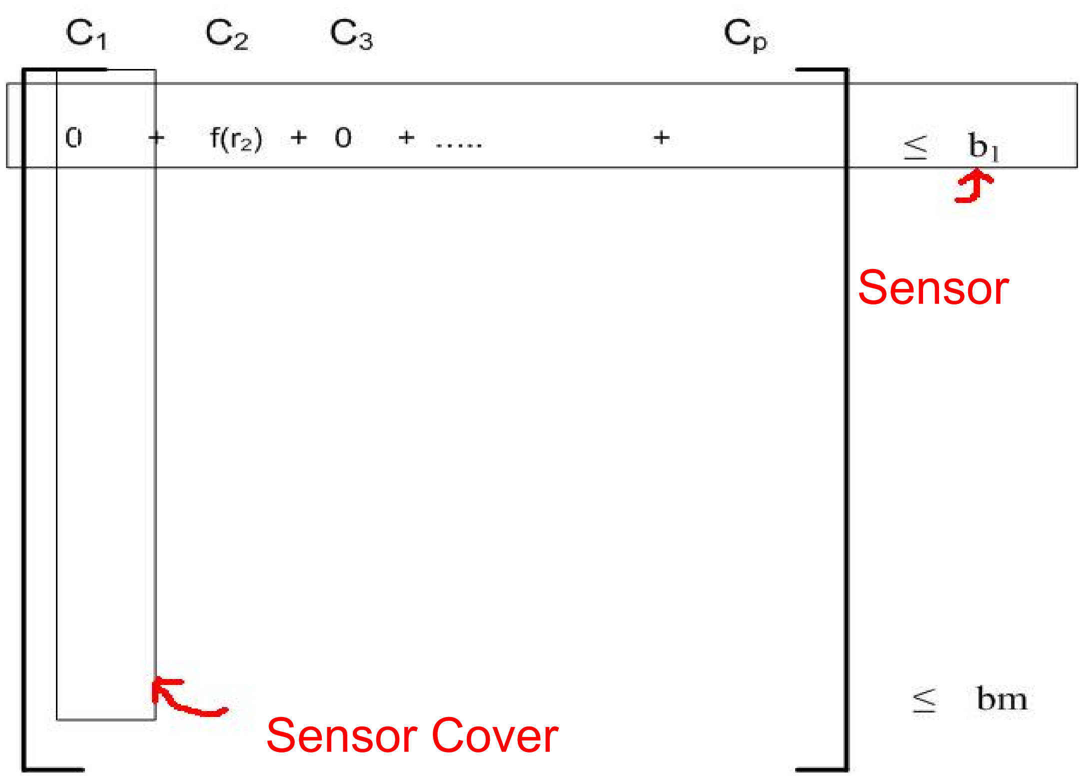
and, $C_{ij} = 0$ if sensor i is not in sensor cover j,

$C_{ij} = g(d)$, if sensor i is in sensor cover j with a sensing range fixed to d and g is a function of energy over distance.

Example

- Suppose *m* sensors, *p* covers
- *Maximize:* $\sum^{p} tj$

Subject to:



Comments

- Substantially different from formulation in *[4]* (Max . $C_1 + C_2 + \ldots + C_k$)
- They indirectly maximize number of sets up to some limit *k*. We directly maximize lifetime *t*
- Also, it can be shown that having more than *n* covers C_j with non-zero t_j is of no use, where *n* is the order of sensors
- Problem: Exponential columns in *n*

Garg-Könemann

- Def^n 1 – Packing LP.
- General form:

$$max\{c^T x \mid Ax \leq b, x \geq 0\} \qquad [8]$$

where, A, b and c are (m x n), (m x 1) and (n x 1) matrices whose entries are positive.

- GK needs an *f*-approximation to finding the minimizing length column of A
- $length_y(j) = \sum_i A(i,j)\, y(i) / c(j)$ for any positive vector y

The algorithm

Input: A vector $b \in R^m$, $\varepsilon > 0$, and an f-approximation algorithm F for the problem of finding the minimum length column $A_{q(y)}$ of a packing LP $\{\max\ c^T x | Ax \le b, x \ge 0\}$

Output: A set of columns $\{A^j\}_{j=1}^k$ each supplied with the value of the corresponding variable x^j, such that $(x^1, \dots, x^k)$ correspond to all non-zero variables in a near-optimal feasible solution of the packing LP $\{\max\ c^T x | Ax \le b, x \ge 0\}$

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(1) **Initialize:** $\delta = (1+\varepsilon)((1+\varepsilon)m)^{-1/\varepsilon}$, for $i = 1, \dots, m$ $y(i) \leftarrow \frac{\delta}{b(i)}$, $D \leftarrow m\delta$, $j = 0$

(2) While $D < 1$

Find the column A_q using the f-approximation F.

Compute p, the index of the row with the minimum $\frac{b(i)}{A_q(i)}$

$j \leftarrow j+1$, $x^j \leftarrow \frac{b(p)}{A_q(p)}$, $A^j \leftarrow A_q$

For $i = 1, \dots, m$, $y(i) \leftarrow y(i)\left(1 + \varepsilon \frac{b(p)}{A_q(p)} / \frac{b(i)}{A_q(i)}\right)$, $D \leftarrow b^T \mathbf{y}$.

(3) Output $\{(A^j, \frac{x^j}{\log_{1+\varepsilon} \frac{1+\varepsilon}{\delta}})\}_{j=1}^k$

Result

- THEOREM 4.1. *The Lifetime problem with adjustable sensing range assignment can be approximated within a factor of $(1+\epsilon)$ f, for any $\epsilon > 0$ by using the Algorithm of Fig. 1, where f is the approximation ratio of the algorithm that picks the minimum weight column in Fig. 1.*

  This result is implied by the Garg-Könemann algorithm [8].

- So we need an *f-approximation* to the dual problem

Minimum Weight Sensor Cover with Adjustable Sensing Range

- Given a monitored region R, a set of sensors $s_1, s_2, \ldots, s_n$ and a set of targets covered by each sensor for a range r_i and the weight w_i for each sensor, find the sensor cover with minimum total weight.
- So the range influences the weight
- *Basic idea:* A sensor wants the best ratio of targets covered to energy spent

A greedy algorithm

Our *f*-approximation is a greedy heuristic that tries to add sensors to the set cover by picking a sensor s_i with a sensing range r_i that maximizes the following ratio:

$$Gval(s_i) = \frac{\text{No. of uncovered targets covered by } s_i}{\text{weight x } e_i}$$

Here, weight is the packing LP variable and is updated by Garg-Könemann. Also, e_i is a function of the distance d_{ij} between sensor s_i and target t_j and can be varied to study linear, quadratic and other energy models.

1. For each sensor s_i compute the vector D_i given below

$$D_i = [\, 1/e_{i1}, \ldots, m/e_{im}]$$

Here, the numerator represents number of targets covered, and, m is the number of targets it can cover with range set to MAXDIST

2. Find the maximum value of D_i

3. Divide D_i / *weight*

weight is the variable from Garg-Könemann for the next step

4. Insert D_i /*weight* into a heap, along with (m_i, r_p) which represents number of uncovered targets and the sensing range respectively.

5. Extract $P=\max(m_i^*, r_p^*)$ from the Binary Heap

6. Update Binary Heap

    for each target t_j covered by P
        for each sensor i in the Binary Heap
            for each $\alpha \in D_i$ vector of that sensor
                if $(d_\alpha \geq d_{ij})$ then
                    $M_\alpha = M_\alpha - 1$
                    //reduce uncovered targets
                else
                    break

7. Update max, rebuild Heap

8. Repeat 2-7 until all targets are covered

Approximation Ratios

- THEOREM 5.1. *The Greedy Algorithm for the Minimum Weight Sensor Cover Problem with Adjustable Sensing Ranges has an approximation ratio $(1+\ln k)$.*

  This is from the standard greedy algorithm for the Minimum Weight Set Cover Problem with k points to cover.

  COROLLARY 5.2. *The Lifetime problem with adjustable sensing range assignment can be approximated within a factor of $(1+\epsilon)(1+\ln m)$ for any $\epsilon > 0$ by using the Algorithm of Fig. 1.*

  This result comes from Theorem 4.1 and Theorem 5.1 with $k=O(m)$ elements to cover, m being the number of targets.

Experimental Results

- 100mx100m area
- Number of Sensors N: 80 to 200
- Number of targets: 25 or 50
- Range *r* : 5m to 60m
- Same as [4] but we allow range to vary smoothly instead of discrete steps
- Same energy models – linear and quadratic
- Use GK to find sensor covers. Then solve LP for assigning time to each sensor cover.

Results

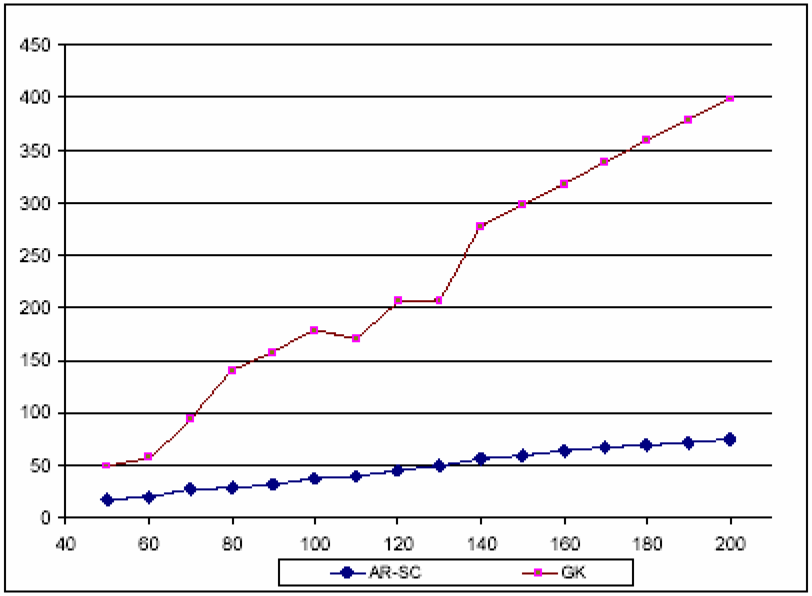


Fig 3. Variation in Network Lifetime with Number of Sensors. Number of Targets=25, Energy model is linear. AR-SC denotes the algorithm in [4]

Results

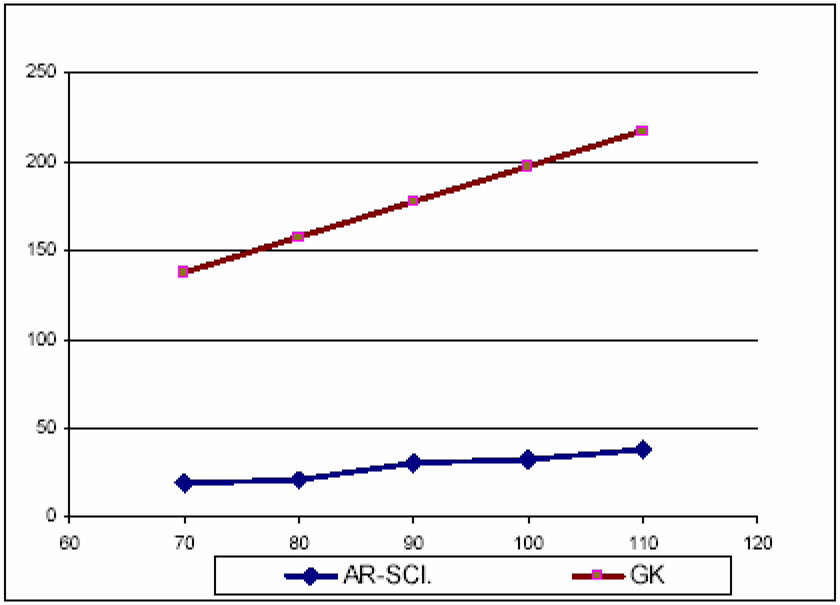


Fig 4. Variation in Network Lifetime with Number of Sensors. Number of Targets=50, Energy model is linear.

Results

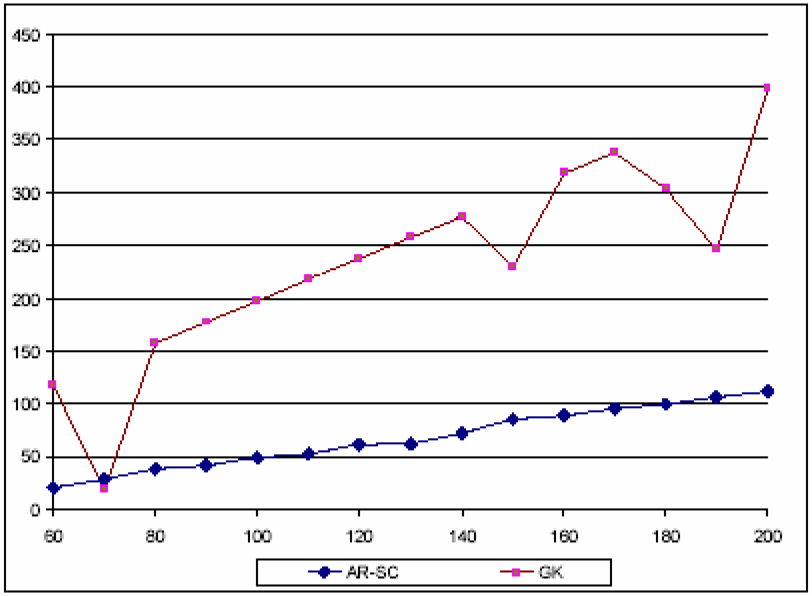
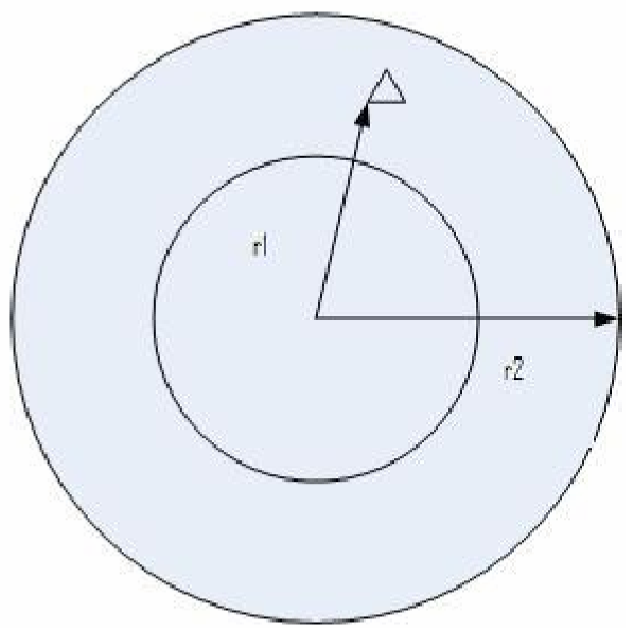


Fig 5. Variation in Network Lifetime with Number of Sensors. Number of Targets=25, Energy model is quadratic.

Reasons for improvement

- Smoothly varying sensing range
- Hence, we spend energy needed to reach target and not the next step



Reasons for improvement

- Ability to assign fractional time to each cover instead of running algorithm in steps
- Provably good algorithm with approximation ration *(1+ln m)*

Conclusions

- New formulation
- Provably good heuristic
- Initial results indicate significant improvement
- Future work – more comparisons, distributed algorithm for the same problem