CSE 462 Randomized Algorithms

Spring 2007, Homework 2, Due on April 18, 2007

1. Present a CRCW PRAM algorithm for finding the maximum of n given numbers in $O(1)$ time using $n^{1+\epsilon}$ processors, where ϵ is any constant > 0.

2. Input are three polynomials $f(x), g(x)$, and $h(x)$. Polynomials $f(x)$ and $g(x)$ have degree n each and $h(x)$ is of degree $2n$. The problem is to check if $f(x)g(x) = h(x)$. Present a Monte Carlo algorithm for this problem that runs in $O(\log n)$ time using $\frac{n}{\log n}$ CREW PRAM processors. Prove that the run time of your algorithm is correct with high probability.

3. Input is a sequence of n numbers (not necessarily in sorted order). The problem is to compute the right neighbor of each element in the sorted order. For example if the input is $6, 12, 5, 3, 17, 11$, the output will be $11, 17, 6, 5, \infty, 12$. Present a Las Vegas algorithm for this problem that runs in $\widetilde{O}(1)$ time. You can use up to n^2 CRCW PRAM processors.

4. Array A is an almost sorted array of n elements. It is given that the position of each key is at most a distance of d away from its final sorted position where d is a constant. Give an $O(1)$ time n-processor EREW PRAM algorithm to sort A. Prove the correctness of your algorithm using the zero-one principle.

5. Present an $O(\log n)$ time algorithm to compute the FFT of a given vector of length n. You can use up to n CREW PRAM processors. As a consequence present a parallel algorithm to compute the product of two given polynomials.

6. Suppose that in a p-processor linear array at most k packets originate from any processor and at most k packets are destined for any processor. Show how to perform this routing task in time $\frac{(k+1)p}{2}$ or less.

7. We can define a 2D mesh with fixed buses as an $n \times n$ mesh where there is a bus for each column and each row. Present a randomized algorithm to perform selection on this model in $\widetilde{O}(n^{1/3})$ steps.